Using the State-Space and Transfer Function Blocks in Simulink

INTRODUCTION

In this tutorial, two additional methods for modeling differential equations in Simulink will be discussed. The state-space and transfer function methods offer a more succinct way of modeling systems and are often used in controls analysis. To better understand their use, the second-order, single-degree of freedom (SDOF) system will be modeled using both techniques.

MODELING WITH THE STATE-SPACE BLOCK

The state-space method is convenient for breaking down a higher-order differential equation into a series of first-order equations for easier solution by matrix methods. To begin, select the State-Space block from the ***Continuous*** sub-menu of the Simulink library. Complete the model with the Step and Scope blocks as shown in Fig. 1.

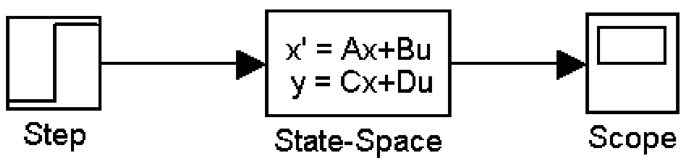

Fig. 1. Basic system model using the State-Space block.

At this point the model is very general, and an equation of any order can be set up for solution in the block parameters. The equation inside the State-Space block is

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}u \\ \mathbf{y} &= \mathbf{C}\mathbf{x} + \mathbf{D}u \end{aligned} \qquad (1)$$

This represents the basic state-space equation, where

$\dot{\mathbf{x}}$ = a vector of the first-order state variables,
$\mathbf{y}$ = the output vector,
$\mathbf{x}$ = the state variable vector,
u = the forcing function,
A = the state matrix,
B = the input matrix,
C = the output matrix, and
D = the transmission matrix.

The second-order ODE that will be modeled is

$$m\ddot{x} + c\dot{x} + kx = f(t)\,. \qquad (2)$$

The first step in putting this into state-space form is to break down this equation into two first-order state equations. This can be done by assigning a subscripted variable for each "state" of the system in the order of increasing derivatives (i.e. x_1 = displacement state and x_2 = velocity state). Therefore, the equations obtained are:

$$x_1 = x = \text{displacement state}$$
$$x_2 = \dot{x} = \dot{x}_1 = \text{velocity state} \qquad (3)$$
$$\dot{x}_2 = \ddot{x}$$

This allows a re-statement of (2) in terms of state variables only. For this example, two first-order differential equations are created.

$$\begin{aligned} \dot{x}_1 &= x_2 \\ m\dot{x}_2 + cx_2 + kx_1 &= f(t) \end{aligned} \qquad (4)$$

These can be reorganized to obtain:

$$\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{c}{m} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} + \begin{Bmatrix} 0 \\ \frac{1}{m} \end{Bmatrix} f(t) . \qquad (5)$$

Compare this to (1). The state and input matrices (matrices A and B) needed to define the block parameters have been defined. What remains are the output matrix C and the transmission matrix D. To solve for these, a desired state of interest must be chosen. For this example, the displacement of the system is important. The resulting output equation is

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} + \begin{bmatrix} 0 \end{bmatrix} u . \qquad (6)$$

Note that, for the purpose of this tutorial, the transmission matrix D will be set to 0. The use of the transmission matrix is beyond the scope of this tutorial.

With all of the matrices defined, the data can be entered into the state-space block parameters. As shown in Fig. 2, the matrix dimensions must be maintained.

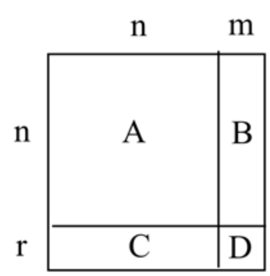

Fig. 2. Required matrix dimensions for State-Space block.

Open the State-Space block parameter window and set the parameters as shown in Fig. 3.

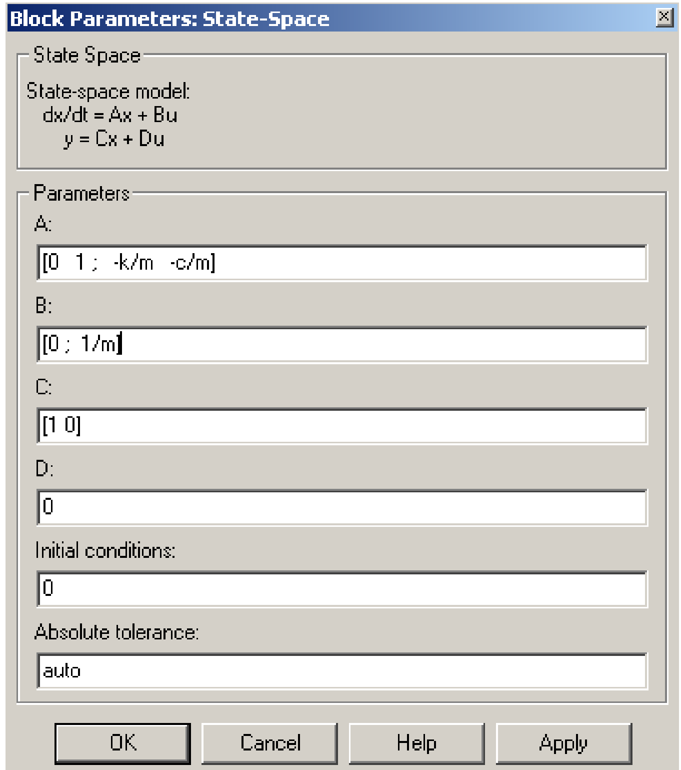

Fig. 3. State-Space Parameter window.

The m, c, and k values shown in the parameter box should be set to

m = 2,
c = 2, and
k = 4.

With these settings, the output will be as shown in Fig. 4.

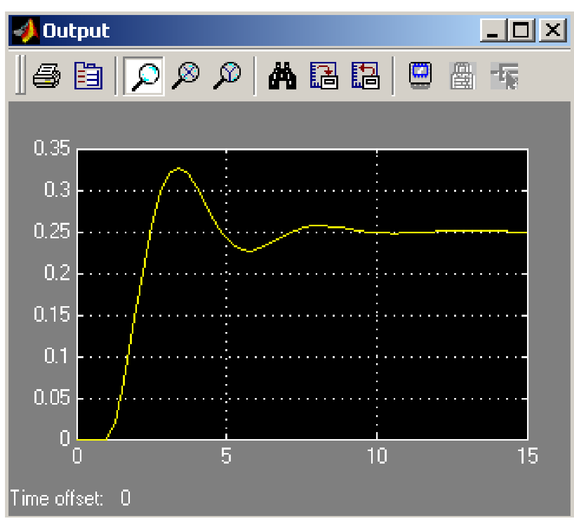

Fig. 4. SDOF response due to a step input using a state-space model.

It is clear that this is an under-damped system. Try changing the m, c, and k values to see how the system will respond with different properties.

MODELING WITH THE TRANSFER FUNCTION BLOCK

The same model will be run using the transfer function block. The advantage of the transfer function block is that it is easy to set up in Simulink. Like the state-space block, all of the information about the system is contained within one block. The system transfer function is essentially a ratio between the Laplace transform of the output response of a system and the Laplace transform of the input forcing function. To obtain the system transfer function for any system, take the Laplace transform of the governing differential equation for that system with all initial conditions set to zero. Then form the ratio of the output response to the input forcing function. For the single-degree-of-freedom system, the basic system transfer function is

$$\frac{X(s)}{F(s)} = H(s) = \frac{1}{ms^2 + cs + k}. \qquad (7)$$

To illustrate the use of the transfer function block, a reevaluation of the example from above will be performed to show that the results will be the same. From the ***Simulink Library Browser***, select ***Continuous***, and drag and drop the Transfer Fcn block into the model space. Also drag a Step block and a Scope block into the model space. Construct the model as shown in Fig. 5.

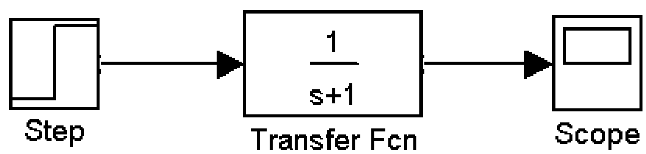

Fig. 5. Basic Transfer Function model.

Double-click on the Transfer Fcn block and set the parameters as shown in Fig. 6.

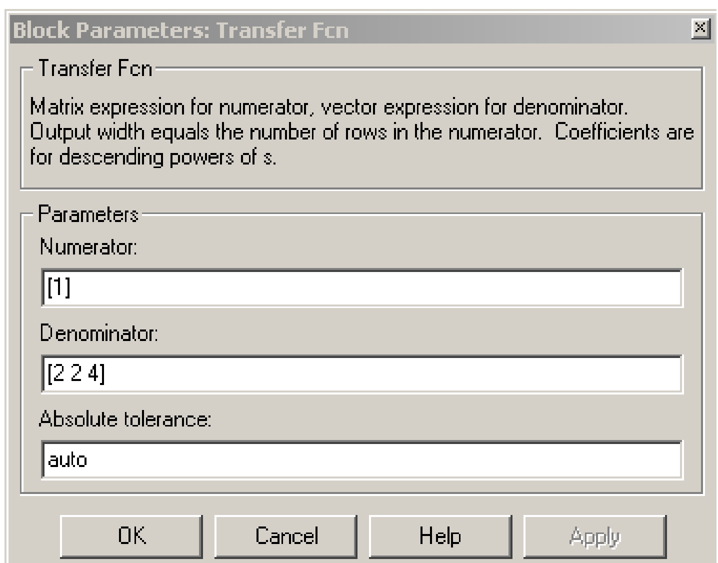

Fig. 6. Transfer function block parameters.

The inputs for this block include the coefficients of the polynomials in the numerator and denominator of the transfer function. Fig. 6 shows how the values from our previous example would be inserted. Selecting ***Apply*** → ***OK***, and the Transfer Fcn block in the model will be updated to show the new values as in Fig. 7

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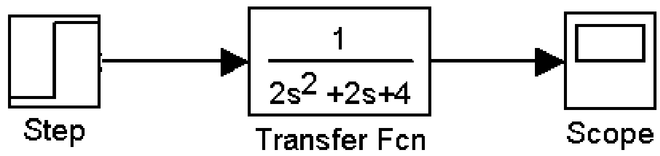

Fig. 7. Specific transfer function model for the SDOF system.

After running the model, a plot that looks like Fig. 8 should be seen.

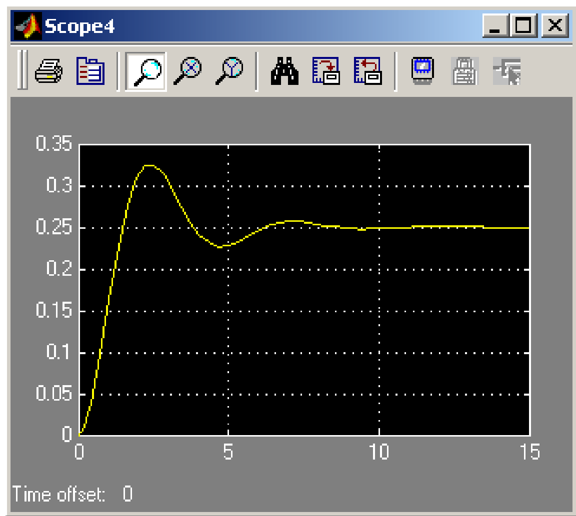

Fig. 8. SDOF response due to a step input using transfer function model.

A comparison of this plot with the one obtained from the state-space model will show that they are identical, as expected. It is apparent that the transfer function model is the easier of the two to use, and it therefore gets wide use in application. It is important that the student be able to obtain the system transfer function for a given differential equation and model the system in Simulink.